Mean Value theorem & extremal values

November 4, 2016

Problems

Problem 1. Does the function $f(x) = x^{\frac{2}{3}}$ satisfy the Mean Value Theorem on the interval $[-1, 1]$? Why or why not?

Problem 2. Let $f(x) = x(x+1)(x+2)(x+3)(x+4)$. Show that the equation $f'(x) = 0$ has exactly four real solutions.

Problem 3. Let P be the parabola passing through the points $A = (1, 2)$, $B = (0, 0)$ and $C = (2, 0)$. Show that the point $D = (3, -6)$ lies on P. Find a point Q on P the tangent to which is parallel to the chord BD.

Problem 4. Two horses start a race at the same time and finish in a tie. Prove that at some time during the race they have the same speed.

Problem 5. Which cylinder of a given volume has the least overall surface area?

Problem 6. You are inscribing rectangles into the region between the graphs of $y = x^2$ and $y = 4$ so that the sides of the rectangles are parallel to the x and y axes. What is the largest perimeter of such an inscribed rectangle you can get?